ABSTRACT
A simplified method developed for the analysis of the mechanical behavior of a building is presented. This method is based on modal analysis of the structure. It relies on a mathematical expansion of the dynamical equations of the movement on the eigenshapes of the structure. The time-space loading is defined by empiric formulae which take into account the weight of equivalent TNT, the distance between the explosive and the structure, and allows to describe the propagation of the shock wave on the structure. Three typical loadings are investigated: long distance explosions (blast loading), medium and near distance explosions. The results concerning the movement of the structure given by the simplified method will be compared to those obtained by a direct time integration method. Particularly, the convergence of the simplified method towards the direct time integration method will be investigated in terms of displacements and stresses versus the explosive-structure distance. This method allows to identify the eigenshapes and eigenvalues of the structure which are important to be taken into account to predict the dynamical response of the building. This method is used as a pre-study (elastic mechanics) i.e. as a first step of a detailed dynamical analysis.

INTRODUCTION
French regulations require to study accidental situations including explosions due to hazards or malevolent acts. As far as nuclear facilities are concerned, IRSN has defined that the general objective of the long term program is to establish the behavior of a facility submitted to an internal or external explosion, so as to evaluate radiological releases in the environment and its consequences. The work presented in this paper illustrates a method used to estimate the mechanical resistance of buildings. It deals with the simulation of the behavior of a building made of concrete submitted to a pressure generated by the explosion of several tens of kg of equivalent TNT. The dynamical response of the structure is evaluated by 3D calculations which are made with CASTEM computer code in order to draw abacus giving the couples (mass of explosive, explosion/structure distance) corresponding to the existence of a failure point and a fracture point in the structure. These abacus allow to establish, for a given building, a forbidden area, where no explosive should be laid.
1. THE WAY THE FRENCH STUDIES ARE ACHIEVED

As required by the French regulations, operators must assess the behavior of their facilities when aggressed by terrorists. The French competent authority provides the operators with the threats to be considered. The aim of these assessments is to determine up to what extent the facility is protected. The technical study of malevolent acts requires two stages: the sensitivity study and the vulnerability study. The sensitivity study aims to evaluate the consequences in terms of safety, radiological release on the environment and on people, of the loss of components, buildings or circuits. Consequences are compared with those considered in the safety report of the facility. This study allows the determination of the most sensitive part and the most critical components of the facility. The vulnerability aims to characterize the difficulty to perform the aggression: difficulty to proceed in the facility, kinds and quantity of tools, weapons and explosive to carry out the aggression. This last aspect leads to assess the mechanical resistance of the structure in case of such an aggression. The needed tools and weapons will be compared with those considered in the threats. The role of IRSN is to provide authority with elements of appreciation and expertise of the studies carried out by the operators.

2. DESCRIPTION OF THE LOADING

As opposed to dedicated computer codes, the CASTEM general computer code developed by the CEA is not able to calculate the propagation of the shock wave. Therefore, the spatial distribution of the shock pressure is given by classical semi-empirical formula [1]:

The free obstacle pressure \( P(M, t) \) is a function of the distance between the structure and the explosive device and a function of the mass of the explosive device, through the following parameters \( t_a, t_d, \alpha, \Delta P_0 \). \( P(M, t) \) is the pressure that can be measured by sensors (without any obstacle for the propagation of the shock wave):

\[
P(M,t) = P_{\text{am}} + \Delta P_0 \left( 1 - \frac{\alpha t - t_d}{t_a - t_d} \right) \exp \left( -\alpha \frac{t - t_d}{t_a - t_d} \right) (t - t_a)
\]

In (Eq. 1):
- \( t_a \) is the arrival time of the shock wave pressure on the building,
- \( t_d \) is the duration time of the shock wave pressure. They are given by abacus tabulated by Kinney and Graham [1].
- \( \Delta P_0 \) is the static overpressure: it is the difference between the maximal pressure observed and the ambient atmospheric pressure, in the lack of obstacle.

In the case of chemical explosions, it is calculated with the expression:

\[
\Delta P_0 = \frac{808}{\sqrt{1 + \frac{z^2}{4.5}}} \left( 1 + \frac{z}{0.048} \right) \left( 1 + \frac{z}{0.32} \right) \left( 1 + \frac{z}{1.35} \right)
\]

(Eq. 2)

All these parameters are expressed through one parameter which only depends on the distance between the structure and the explosive device, and the mass of equivalent TNT of the explosive substance.

\[
z = \frac{d}{W^{0.75}}
\]

is the reduced distance [1].

The reflected pressure \( P_r(t,M) \) is expressed, using the free obstacle pressure \( P(M,t) \), by:

\[
P_r(M,t) = P_{\text{am}} + \left( P(M,t) - P_{\text{am}} \right) \left( \frac{\gamma - 1}{\gamma + 1} \right) P(M,t) + \left( \frac{\gamma + 1}{\gamma - 1} \right) P_{\text{am}}
\]

This reflected pressure is used as external load applied on the structure.

These formulae have been compared with the pressure computed by hydrodynamic codes and with pressures obtained by experiments [2] in order to determine their domain of validity. These studies show that the semi-empirical formulae are suitable for reduced distance \( z \) superior or equal to 0.2.

Three types of loading will be investigated: long distance explosions (100 m) (blast loading), medium (10 m) and near distance explosions (1 m).

Although the different parameters depend on the weight of the explosive substance, the ranges of the parameters are given in the table 1.

<table>
<thead>
<tr>
<th>Distance Structure/Explosive device</th>
<th>Time of application (s)</th>
<th>Time of arrival (s)</th>
<th>Running time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 m</td>
<td>From 1.0E-3</td>
<td>From 4.1E-4</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>To 9.5E-3</td>
<td>To 7.1E-2</td>
<td></td>
</tr>
<tr>
<td>10 m</td>
<td>From 9.6E-3</td>
<td>From 1.0E-2</td>
<td>0.052</td>
</tr>
<tr>
<td></td>
<td>To 1.7E-2</td>
<td>To 6.2E-2</td>
<td></td>
</tr>
<tr>
<td>100 m</td>
<td>From 4.10E-2</td>
<td>To 2.3E-1</td>
<td>0.012</td>
</tr>
<tr>
<td></td>
<td>To 4.14E-2</td>
<td>To 2.4E-1</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Value ranges of the parameters for the studied building

The running time of the shock wave corresponds to the time needed to run over the building from the bottom to the top. For long distance explosions, table 1 shows that the time of application of the pressure as well as the time of arrival of the pressure shock wave are constant along the faced wall: for these explosions, the loading can be considered as a uniformed and constant pressure. The propagation shock wave velocity is drawn on fig. 2.
3. DESCRIPTION OF THE BUILDING

In this paper, the studied building is a typical building (in terms of mass, dimensions, thickness of the walls) of nuclear facilities, but, for evident reasons of confidentiality, it does not represent an existing building. The building has a roughly cubic shape. The thickness of the walls and floors varies from \( e_0 \) to 3.25\( e_0 \) m. The building is made of concrete. It is composed of simple elements involved in a nuclear facility (walls, floors, water tanks, additional masses, ...). Figures 3 and 4 illustrate vertical and longitudinal cross section of the building.

The building is modeled using 4-nodes shell elements. Additional masses are used to represent the water contained in the tanks and internal components. The mass of the water stands for 19% of the mass of the building, the additional masses stand for 2% of the mass of the building.

The constitutive law of the concrete is assumed to be linear. The building is modeled using 4-nodes shell elements. Additional masses are used to represent the water contained in the tanks and internal components. The mass of the water stands for 21% of the mass of the building, the additional masses stand for 2% of the mass of the building.

The building is modeled using 4-nodes shell elements. Additional masses are used to represent the water contained in the tanks and internal components. The mass of the water stands for 19% of the mass of the building, the additional masses stand for 2% of the mass of the building. Therefore, as a first step, no structural damping is considered. Openings, doors or windows are neglected. No soil-structure interaction is considered: the building is supposed to be clamped on the ground.

4. DESCRIPTION OF THE MODAL METHOD

The system of equations to be solved is given by (Eq. 2):

\[
\ddot{\mathbf{X}} + \mathbf{m} \ddot{\mathbf{X}} + \mathbf{k} \mathbf{X} = \mathbf{F} \quad \text{with} \quad \mathbf{F} = \int_S \mathbf{P}(\mathbf{m}, t) \mathbf{n} \, ds \quad \text{(Eq. 2)}
\]

\( \mathbf{F} \) is the force resulting from the overpressure applied on the building.

To solve this system of equations, two methods can be applied: the direct time integration method and the modal method.

The direct time integration method consists in solving equation (Eq. 2) at each time step. It is simple to use. It can be used for non-linear calculations and allows to study what happens locally. Nevertheless, parametric studies are cost consuming because new complete calculations are necessary for each value of investigated parameters.

The modal method consists in expanding the movement of the structure on elementary shapes (the eigenshapes) \( \phi_i \) which are solutions of the equations \( (\mathbf{k} - \omega_i^2 \mathbf{m}) \phi_i = 0 \). So, the displacement can be written:

\[
\dot{\mathbf{X}}(\mathbf{m}, t) = \sum_{i=1}^{\text{number of modes}} \alpha_i(t) \phi_i(\mathbf{m}) \quad \text{(Eq. 3)}
\]

Replacing \( \dot{\mathbf{X}}(\mathbf{m}, t) \) by its expression (Eq. 3) in (Eq. 2), and using the normal properties of the eigenshapes [3], leads to:

\[
\ddot{\alpha}_i + \omega_i^2 \alpha_i = \frac{1}{m_{\text{q}}^*} \left( \mathbf{\phi}_i^T \mathbf{F} \right) \quad \text{(Eq. 4)}
\]

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\[
\ddot{\alpha}_i + \omega_i^2 \alpha_i = \frac{1}{m_{\text{q}}^*} \left( \mathbf{\phi}_i^T \mathbf{F} \right) \quad \text{(Eq. 4)}
\]

The advantage of this method is to identify the eigenmodes which contribute the most significantly to predict the dynamical response of the building. Parametric studies are easier: the modal basis is computed only once. The time integration is made easier thanks to diagonal mass matrix.

The main limitation of this method is that it does not allow to simulate non-linear mechanical behaviors. Moreover the computation of the eigenmodes is based on the use of a finite element model (i.e. with a limited number of DOF). As a consequence, only the first modes with the lowest frequencies can be correctly calculated and have a physical significance.

Dynamic calculations are performed with a truncated basis. The computation of the modal basis may also be difficult to achieve when the basis contains modes with eigenfrequencies close one to the other. It is worth also to point out that similar effects of truncation exist when using finite elements model in a direct time integration scheme. They are solved thanks to specific time integration algorithms.

The truncation criterion of the modal basis request a specific attention. The truncated modal basis must contain a sufficient number of modes:

- to represent the frequencies involved within the load: when the load has a small time duration, high frequencies are required.
- to represent the spatial aspects of the load: when the detonation point is far from the structure, the structure will have a global movement (low frequency behavior); whereas when the detonation point is close to the structure, the movement of the structure will correspond to local modes (high frequency modes).
- to study the physical phenomena and parameters of interest

Therefore, the choice of the truncated basis must be a compromise between the needed accuracy and the calculation time. Whatever the method is used, the assumption of small disturbances is done.

5. MODAL ANALYSIS

The eigenshapes and eigenvalues are searched within an interval of frequencies from 0 Hz to 125 Hz. The first eigenvalues \( f = 9.6 \) Hz is a global bending movement of the building in the x-direction. The second eigenvalues \( f = 10.9 \) Hz is a global bending movement of the building in the y-direction. The third eigenvalues \( f = 14.6 \) Hz corresponds to a torsional movement of the building around the z-axis, due to the mass and stiffness dissymmetry of the structure.

The other eigenvalues correspond to local bending movement of a wall or a floor.

By computing \( \sum_{i=1}^{\text{number of modes}} \frac{\alpha_i^2}{m_i} \) for each direction, the fraction of the building total mass involved in the eigenmodes contained in the modal basis can be expressed.

It is worth pointing out that \( \frac{\alpha_i^2}{m_i} \) does not depend on the way the eigenshapes are normalized.

The fraction mass ratio depends on the frequency range covered by the modal basis and can be used as a criterion to
check the quality of the truncated basis since it represents the kinetic energy of the system [3].
In the following computations, three modal basis have been considered:

Using 35 modes (for \( f \in [0;37\,\text{Hz}] \)), the fraction of the total mass represented is:
- 80\% of the total mass in the x-direction
- 80.7\% of the total mass in the y-direction
- 72\% of the total mass in the z-direction

Using 100 modes (for \( f \in [0;60\,\text{Hz}] \)), the fraction of the total mass represented is:
- 82\% of the total mass in the x-direction
- 82\% of the total mass in the y-direction
- 78\% of the total mass in the z-direction

Using 354 modes (for \( f \in [0;125\,\text{Hz}] \)), the fraction of the total mass represented is:
- 84.3\% of the total mass in the x-direction
- 84\% of the total mass in the y-direction
- 82.5\% of the total mass in the z-direction

6. COMPARISON OF THE DIFFERENT METHODS

The results concerning the movement of the structure given by the simplified method are compared to those obtained by a direct time integration method. Due to the blast loading, the wall standing just in front of the starting point of the explosion is undergoing maximal stresses. The maximal stresses are located at the embedding, at wall-floor links or at variation of section of the wall, depending on the structure/explosion distance.

The displacements of the building are over few millimeters. For the following results, the explosive device is placed face to the wall V09.

In a first stage, the convergence of the modal basis towards the exact solution has been investigated in considering the most complete basis (i.e. 354 modes up to 125 Hz) as a function of the distance of the explosive source from the building. Figures 5,7 and 9 show the good agreement of the two kinds of methods (direct integration and modal method) whatever the distance explosive-device/structure is studied, as far as displacements are the relevant results.

The results are similar for the stresses (See Fig. 6 for the Von Mises Stress), using the modal basis for \( f \in [0;125\,\text{Hz}] \), and for long distance explosions.

Nevertheless, this modal basis is not sufficient to compute the stresses with accuracy when the detonation occurs close to the building (i.e. when the contribution of high order modes increases): an error of 10\% (see fig. 8) is made for a 10 meters explosion, whereas the error reaches 35\% (see fig. 10) for a 1 meter explosion.

More generally, it can be showed by analytical calculations that the method using modal projection converges more slowly for stresses than for displacements towards the exact solution assumed to be close to the results obtained with the direct time integration method.

The convergence of the modal method towards the direct time integration method has been also investigated in terms of displacements and stresses versus the explosive-structure distance as a function of the number of modes in the modal basis.

Figure 11 shows the convergence of the displacements computed by the two methods for long distance explosions. It can be observed that when the number of modes in the modal basis is greater than 100 (i.e. for a cut-off frequency greater than 60 Hz) a good agreement between the two methods is observed. With only a cut-off frequency equal to 37 Hz, significant discrepancies (in amplitude and in shape) may be observed between the two methods. It can be explained in noticing that the dominant frequency in the exact response is around 39 Hz, which is slightly greater than the cut-off frequency in this case. Similar conclusion can be made for the stresses (figure 12) even though the discrepancies seem to be less important which is related with the fact that the significant contributions to the stresses do not come from the lowest modes.

The results are quite similar for medium distance explosions (figures 13 and 14) but the discrepancy of the stresses is more pronounced.

For short distance explosions, the convergence is hardly acceptable for the stresses even with 354 modes which characterizes the important contribution of high order modes. As previously mentioned, one advantage of the modal synthesis method is to determine which eigenmodes are predominant within the mechanical response of the structure. This aspect allows a better understanding of the physical phenomena involved in the response.

The solutions \( \alpha_i \) of (Eq. 4) being computed, each \( \alpha_i \phi_i \) is normalized to the maximum value of \( \alpha_i \phi_i \) (The values of \( \alpha_i \phi_i \) are given at the observed point P9).

Therefore, we can draw – as shown on fig. 17 to 22 – the weight of each eigenshape within the response of the structure (for the displacements and the stresses).

It can be observed that even for long distance explosions, the dominating eigenmode is not the one which corresponds to a global bending movement of the whole structure within the direction of the shock wave propagation.

For every distance explosion presented in this paper, the dominating eigenmode corresponds to a bending mode of the wall facing the explosive device.

Figures 18 and 19 show the increasing weight of higher frequency modes. This increasing weight appear in number of modes as well as in intensity.

Figure 20 represents the participation of each eigenmode for stresses for a 100 meters explosion. The frequency of the predominant eigenmode (\( f = 20.3\,\text{Hz} \)) is included in the modal basis using 35 modes, as shown on fig. 12. This truncated modal basis is sufficient to represent the stresses with an acceptable accuracy.

For 10 meters explosions, the frequency of the predominant eigenmode (\( f = 39.1\,\text{Hz} \)) is not included in the modal basis using 35 modes (fig. 21). The cut-off frequency is too low for this truncated modal basis. Nevertheless, the second truncated modal basis contains enough eigenmodes to represent the stresses (as well as the displacements) with an acceptable accuracy.

On the counterpart, for short distance explosions, high order eigenmodes cannot be neglected as shown on fig. 22, which explains the discrepancies observed on fig. 16.

CONCLUSION

The modal synthesis method has been used to compute the linear mechanical behavior of a building submitted to an external detonation of hundreds of kilograms of TNT. The validity of this approach has been proved through a comparison to a direct time integration method.

The effect of the truncation of the modal basis has been especially investigated.

The modal synthesis allows to easily determine the couples (mass of explosive, explosion/structure distance) corresponding to the existence of a failure point (\( \approx 3\,\text{MPa} \)).
and a fracture point (= 30 MPa) in the structure. These couples allow to draw abacus (figure 20). Below the 3MPa-curve, no failure is to be feared.
Such abacus could be provided to the competent authority to appreciate the security of a facility and take the necessary disposals.

The future works will concern two different ways : the improvement of the determination of the truncation criterion in order to limit as much as possible the size of the modal basis and to take into account the nonlinear behavior of the building, phenomenon which may be significant in the case of strong excitation.

REFERENCES
Figure 5: Displacement of the point P9 (located on the wall V09 (see figure 2))
- Distance structure/Explosive device = 100 m
- Comparison of the 2 methods (354 modes)

Figure 6: Maximal Von Mises Stress
- Distance Structure/Explosive device = 100 m
- Comparison of the 2 methods (354 modes)

Figure 7: Displacement of the point P9
- Distance structure/Explosive device = 10 m
- Comparison of the 2 methods (354 modes)

Figure 8: Maximal Von Mises Stress
- Distance Structure/Explosive device = 10 m
- Comparison of the 2 methods (354 modes)
Figure 9: - Displacement of the point P9 -
- Distance structure/Explosive device = 1 m -
- Comparison of the 2 methods (354 modes) -

Figure 10: - Maximal Von Mises Stress –
- Distance Structure/explosive device = 1m –
- Comparison of the 2 methods (354 modes) -

Figure 11: - Displacement of the point P9 -
- Distance structure/Explosive device = 100 m -
- Comparison of the different modal basis -

Figure 12: - Maximal Von Mises Stress –
- Distance Structure/explosive device = 100 m –
- Comparison of the different modal basis -
Figure 13: Displacement of the point P9 -
- Distance structure/Explosive device = 10 m -
- Comparison of the different modal basis -

Figure 14: Maximal Von Mises Stress -
- Distance Structure/explosive device = 10 m -
- Comparison of the different modal basis -

Figure 15: Displacement of the point P9 -
- Distance structure/Explosive device = 1 m -
- Comparison of the different modal basis -

Figure 16: Maximal Von Mises Stress -
- Distance Structure/explosive device = 1 m -
- Comparison of the different modal basis -
Figure 17: Distribution of the participation of each eigenshape for displacements for a 100 meters explosion

Figure 18: Distribution of the participation of each eigenshape for displacements for a 10 meters explosion

Figure 19: Distribution of the participation of each eigenshape for displacements for a 1 meter explosion

Figure 20: Distribution of the participation of each eigenshape for stresses for a 100 meters explosion

Figure 21: Distribution of each eigenshape for stresses for a 10 meters explosion

Figure 22: Distribution of each eigenshape for stresses for a 1 meter explosion